Examination of Mathematics Teachers' Pedagogical Content Knowledge of Probability

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ABSTRACT

The aim of this study is to explore the probability-related pedagogical content knowledge (PCK) of secondary school mathematics teachers in terms of content knowledge, curriculum knowledge, student knowledge, and knowledge of teaching methods and strategies. Case study design, a qualitative research model, was used in the study, and the participants were three secondary school mathematics teachers. Data collected via observations and semi-structured interviews were analyzed using a deductive approach. Findings indicate that the PCK of these secondary school mathematics teachers about probability is insufficient; furthermore, teachers' beliefs were the most important factors impacting their PCK. In addition, one of the results is that professional experience has a partial effect on PCK.

Keywords: *Mathematics education, mathematics teachers, pedagogical content knowledge, probability.*

INTRODUCTION

Many studies have examined factors affecting academic achievement in mathematics courses from pre-school to higher education (e.g., Bean, Bush, McKenry, & Wilson, 2003; Boozer & Rouse, 2001; Goddard, Sweetland, & Hoy, 2000). Of course, teacher quality is one of the most important variables in academic achievement (Darling-Hammond, 2000; Ojimba, 2013; Park & Hannum, 2002; Rivkin, Hanushek, & Kain, 2005; Rockoff, 2004); several studies have demonstrated the impact of effective teachers on students' achievement (Hanushek, 1992; Rivkin et al. 2005; Sanders & Rivers, 1996; Schacter & Thum, 2004). Thus, qualified teachers should have very good pedagogical knowledge, as well as knowledge of mathematics (Darling-Hammond & Sykes, 2003).

The Background of This Study

Shulman (1986), who has pioneered many studies featuring the kind of knowledge that teachers should possess to provide effective instruction, first addressed teacher knowledge in three dimensions: content knowledge, curriculum knowledge, and pedagogical content knowledge (PCK). Later, he developed a new model by increasing the number of these components (Shulman, 1987). According to Shulman (1987), seven categories of teacher knowledge are (*i*) subject matter knowledge; (*ii*) general pedagogical knowledge; (*iii*) pedagogical content knowledge; (*iv*) knowledge of learners and learning; (*v*) curriculum knowledge; (*vi*) knowledge of educational contexts; and (*vii*) knowledge of educational philosophies, goals, and objectives. After Shulman, many researchers have debated about the components of teaching knowledge and have proposed various models (Cochran, DeRuiter, & King, 1993; Grossman, 1990; Ma, 1999; Smith & Neale, 1989, Tamir, 1988). Tamir's model (1988) includes student knowledge, curriculum knowledge, knowledge of teaching strategies, knowledge of assessment, subject-matter knowledge, and knowledge of pedagogy, while Grossman's model (1990) includes conceptions of purposes for teaching subject matter, knowledge of students' understanding, curriculum knowledge, and knowledge of instructional strategies. Smith and Neale (1989) formed PCK including four components, which are knowledge of students, knowledge of teaching strategies, knowledge of shaping and elaborating the content, and knowledge of curriculum materials. Marks (1990) organized the PCK as consisting of subject-matter knowledge, knowledge of students' understanding, media for the instruction, and instructional processes. Fennema and Franke (1992) categorized PCK in three main components, which are knowledge of students, knowledge of instructional strategies, and subject-matter knowledge. Magnusson, Krajcik, and Borko (1999) proposed a model of PCK including the components of knowledge of purposes, knowledge of curriculum, knowledge of students' understanding, knowledge of instructional strategies, and knowledge of assessment. An, Kulm, and Wu (2004) and Hill, Ball and Schilling (2008) formed similar models including student knowledge, curriculum knowledge, subject-matter knowledge, and knowledge of pedagogy. Hashweh (2005) proposed a more comprehensive model consisting of knowledge of purposes, knowledge of learners, curriculum knowledge, knowledge of instructional strategies and resources, knowledge of measurement and assessment, content knowledge, knowledge of context, and knowledge of pedagogy. Student knowledge takes place in almost each model, and knowledge of teaching methods and strategies is necessary for the students' academic achievement. Additionally, content knowledge is indispensable for effective teaching. Curriculum knowledge and knowledge of instructional strategies are the other two components taking place in the models. As the curriculums have been updated in 2005 and 2013 in Turkey, it is important to examine teachers' familiarity with the entire curriculum. Benefiting from all these models about PCK, we adopted a holistic and pragmatic view in order to investigate the knowledge of the teachers rather than embracing just one approach. In this context, the four components considered to be at the focus of PCK, i.e., content knowledge, curriculum knowledge, student knowledge, and knowledge of teaching methods and strategies, are examined within the present study.

Table 1: The Componenents of PCK from Different Conceptualizations

	Subject Matter Knowledge	Knowledge of Students' Understanding	Knowledge of Curriculum	Knowledge of Instructional Strategies	Conceptions of Teaching Subject	Knowledge of Assessment	Knowledge of Context
Shulman (1987)	~	~	~	~	~	~	~
Tamir, 1988	~	√	√	√			
Smith & Neale (1989)	√	√	√	√			
Grossman (1990)	~	√	√	√	√		~
Marks (1990)	√	√	√	√		√	
Fennema and Franke (1992)	√	√		√			
Cochran, DeRuiter, & King (1993)	√	√	√	√			√
Magnusson, Krajcik, & Borko (1999)	~	√	√	√	√	√	
An, Kulm, & Wu (2004)	√	√	√	√			
Hashweh (2005)	√	√	√	√		√	√
Hill, Ball and Schilling (2008)	√	√	√	√			

- Content knowledge, which is the basic component of teaching knowledge (Ball & McDiarmid, 1990) and the essential knowledge impacting student achievement (Even, 1990; Kim, 2007; National Council of Teachers of Mathematics [NCTM], 2000), implies the teacher having a deep knowledge about the subject (Shulman, 1986). Teachers' in-depth and accurate information about mathematics increases the effectiveness of teaching (Ball, 1990a; Gess-Newsome, 1999). Being competent in content knowledge helps them know what kind of prerequisite knowledge is necessary to teach a certain subject, what the appropriate examples and homework are, and what kind of illustrations can be used (Shulman, 1986). As described by Hill, Rowan, and Ball (2005), the components of mathematical teaching for knowledge are essentially related to content knowledge.
- Besides having content knowledge, it is also important for teachers to know the relationship of this knowledge with other information in the curriculum and how they complement each other. Shulman (1986, 1987) stated that curricula provide teachers "the tools and methods required to perform their jobs." In addition to knowing the development of the subjects and concepts, as well as the relationship of the topics to each other and to other disciplines, teachers also need to know how the topics are covered in the curriculum in the preceding and following years (Shulman, 1986). Magnusson et al. (1999) claimed that the knowledge of curriculum includes two parts: (i) knowledge about the goals and objectives of the curriculum and (ii) knowledge about special programs in the curriculum. On the other hand, Grossman (1990) proposed that curricular knowledge covers both the knowledge of horizontal and vertical curricula for a subject. Knowledge of horizontal curricula includes the knowledge of other concepts and subjects related to the specific subject within a grade level, while the knowledge of vertical curricula implies the knowledge of related concepts and subjects with the specific subject within all grade levels. Teachers not only need to have content knowledge but also they need to know about the curriculum and its educational goals (An, Kulm, & Wu, 2004; Marks, 1990; Park and Oliver, 2008).
- Another important component that teachers should possess to provide an effective education is student knowledge. Shulman (1986) pointed out that teachers should question what makes the learning of some topics difficult or easy, as well as the concepts that students, who have different backgrounds, bring to the classroom. Student knowledge, which is defined as the information that teachers have about students' procedural and conceptual knowledge, learning process, learning styles, learning difficulties and misconceptions (An, Kulm, & Wu, 2004; Shulman 1987), can also be expressed as the knowledge of the potential difficulties that students may face while learning the subject and the requirements for students to learn the concepts (Magnusson et al., 1999).
- In addition to these kinds of knowledge, it is also important for teachers to know how to transfer mathematical content in a way that students can understand, and thus teachers should have a good strategy and representational knowledge. Shulman (1987) considered the knowledge of teaching strategies and methods as a transformation; in other words, the presentation of the subject in the forms that students can understand. He further discussed them under the headings of knowing the most functional representation of topics and concepts; knowing what facilitates learning the subject or what makes it difficult; and knowing simulations, representations, examples, and explanations for better understanding concepts and eliminating misconceptions. What is important for effective teaching is that teachers look through the window of the students to make mathematical knowledge convenient for them; in other words, to stretch it (NCTM, 2000). Knowledge of instructional strategies also includes learning activities and use of materials in addition to the representations such as explanations, metaphors, examples, illustrations, and analogies that facilitate students' understanding (Grossman, 1990; Marks, 1990; Park & Oliver, 2008).

When we examine the components of PCK, we can say that components are specific to a particular subject matter (Shulman, 1986). Therefore, it is necessary to examine the PCK for the topic-specific. The fact that deep thinking is needed to understand probability, which contributes to the development of students' mathematical reasoning (Gürbüz, 2006), indicates that attention should be paid to this subject. Both teachers and students experience various difficulties during the instruction of probability, and this instruction cannot be performed effectively (Boyacıoğlu, Erduran ve Alkan, 1996; Bulut, Ekici ve İşeri, 1999; Sezgin-Memnun, 2008). Low levels of readiness due to the lack of some basic concepts, the difficulty of learning probability at early ages, students' lack of reasoning skills, teachers' lack of knowledge and experience for probability instruction, and students' various misconceptions and negative attitude can be considered among these difficulties (Sezgin-Memnun, 2008). Other challenges include linguistic difficulties, difficulties in transforming the practice into mathematical structure, and lack of belief for analyzing the chance phenomenon from specific intuitional viewpoints (Gürbüz, 2008). Fischbein (1987) also drew attention to the role of intuition on probability learning. Concepts related to probability cannot be taught enough to the students, which is one of the most important problems of the educational process. Studies featuring this issue are still continuing in Turkey (Gürbüz, 2007; Işık, Kaplan, & Zehir, 2011; Özbek, 2002; Sezgin-Memnun, 2008). The study of probability has distinctive characteristics that are not encountered by teachers and students in other areas of mathematics. Although these distinctive features certainly broaden and strengthen the mathematics curriculum, they also create special challenges for teaching and learning (Jones, 2005). In light of the difficulties experienced on the subject of probability and the issues mentioned in the literature, the examination of teachers' PCK about probability is becoming more important in terms of the instruction's effectiveness.

There are a limited number of researches studying the PCK on probability in the literature. Chick and Baker (2005) investigated two teachers' lessons in fifth grade and concluded that occasion gaps in content and pedagogical content knowledge had the potential to cause misconceptions for students. Wilburne and Long (2010) 70 preservice teachers content knowledge and their results showed that the preservice teachers had the lowest mean scores in data analysis and probability compared to other strands. Lim and Guerra (2013) studied with 29 preservice teachers and concluded that their overall content knowledge was not strong and the two lowest-performing content knowledge areas were Geometry/Measurement and Probability/Statistics. Ives (2007, 2009) found that preservice teachers' understanding of randomness and their perspective of the meaning of probability were related. Another study with preservice teachers by Contreras, Batanero, Diaz and Fernandes (2011) revealed that the participants had poor common and specialized knowledge of elementary probability. Brijlall (2014) studied the written responses to an open-ended questionnaire by 86 ninth grade teachers and came up with three domains (i. e., common content knowledge, specialized content knowledge and knowledge of content and teaching) to identify the pedagogical content knowledge necessary for teaching probability. Papaieronymou (2009) identified the suggested probability knowledge for secondary mathematics teachers analyzing the recommendations from four professional organizations (American Mathematical Society, the American Statistical Association, the Mathematical Association of America, and the National Council of Teachers of Mathematics). The author concluded that 66% of the recommendations related to subject matter content knowledge whereas 24% referred to pedagogical content knowledge and only 10% referred to curricular knowledge. Swenson's (1997) study raised a few conclusions about the PCK of teachers on probability. According to the her results, teachers; (i) lacked an explicit and connected knowledge of probability content, (ii) held traditional views about mathematics and the learning and teaching of mathematics, (iii) lacked an understanding of the "big ideas" to be emphasized in probability instruction, (iv) lacked knowledge of students' possible conceptions and misconceptions, (v) lacked the knowledge and skills needed to orchestrate discourse in ways that promoted students' higher level learning, and (vi) lacked an integrated understanding of the nature of the reform in their probability instruction.

As can be seen in the studies related to the PCK about probability, apart from their rarity, studies mostly conducted on the pre-service teachers or used the written forms of teachers. It can be claimed that just Swenson (1997) studied the topic in question thoroughly almost two decades ago. With reference to this fact, it is important to evaluate the PCK of teachers related to the probability through getting into an interaction with them.

Due to its importance and the complexity of measuring all components of PCK simultaneously, the purpose of the study was to examine the PCK of the secondary school teachers about probability in terms of (i) content knowledge, (ii) curriculum knowledge, (iii) student knowledge, and (iv) knowledge of teaching methods and strategies. The results of this study are considered to contribute to the PCK literature and probability instruction so that the researchers could take these results as a reference to further studies and the teacher training institutions could base their curriculums on the results of such studies. The research questions were as follows:

Within the context of secondary school mathematics teachers' probability-related PCK, how do their i) subject-matter knowledge, ii) curriculum knowledge, iii) knowledge of students, and iv) knowledge of teaching methods and strategies interact with each other?

METHODOLOGY

Research Design

The study used a multiple case study design, a qualitative research model. A multiple case study enables the researcher to explore differences within and between cases (Yin, 2003). Although Merriam (2009) suggested that including multiple cases in a study makes the findings and interpretations more compelling, we used multiple cases of the same phenomenon (probability related PCK) aiming to collaborate, qualify, and extend the findings that might occur. In multiple case study design, it is possible to work on each case that has been considered or covered in the study by dividing it into several sub-units (Baxter & Jack, 2008). The reason for choosing this model is to investigate PCK of three different teachers before, during and after a course.

Participants

In the study, criterion sampling, a purposive sampling method, was used in the selection of participants. The basic understanding of the criterion sampling method is selecting the cases that meet some predetermined criterion of importance (Patton, 2001). In this study, teachers' seniority and not having postgraduate training were identified as the main criteria. Teachers' seniority was determined as a criterion since we wanted to see how the PCK components differ according to seniority. Two teachers having the same seniority were included in the sample with the aim of examining the effect of self-improvement on PCK. Not attending postgraduate education was set as a criterion on the thought that teachers' knowledge about the aforementioned components should be implicit for them and they should not be aware of this particular issue. Participants consisted of three volunteer teachers, one having over 30 years of experience and the other two having around 10 years of experience (*See* Table 2).

Table 2: The Demographic Properties and Professional Development of the Participants

	Esin	**Lale**	**Nihat**
Gender	Female	Female	Male
Seniority	10 years	9 years	34 years
Professional Development	Designing a project Material development, guidance, basic computer training, solving the problems, relating the instruction with daily life, use of smart board	Basic computer training, use of smart board	Basic computer training, use of smart board

Table 2 also shows the professional development of the participants and their in-service trainings. In this context, findings will be discussed on the basis of the differentiation according to participants' efforts to develop themselves and their seniority.

Instruments

Within the study, a semi-structured Preliminary Interview Form developed by the researchers was used to examine the PCK in terms of the identified components. This instrument was used to interview the teachers before starting probability instruction. The interview questions were selected to ensure the measurement of related components of PCK. The form was developed in an interactive process with experts in the field. First, the literature was reviewed and the interview form was created with the contributions of the researchers. The required changes were made according to the suggestions of an expert in mathematics education and qualitative research. For example, the question "Is the subject of probability necessary in secondary school?" has been suggested to change as "Why is the probability important to be learned?". A few questions were brought together and asked in a summarized way and a few questions were included according to the suggestions. A pilot interview was conducted with a postgraduate student to test the understandability of the items in the interview form. Some of the items were revised and changed afterwards. The final interview form included 9 basic and 26 probe questions. Apart from these interviews, one of the researchers took field and observation notes during the lessons.

Data Collection and Procedure

Before the lecture, a preliminary 40- to 45-minute interview about the context of PCK components was conducted with the teachers using the semi-structured interview form, and various questions were addressed, such as how teachers perceive the concept of probability, how they plan to give the lecture, what they think about students' prior knowledge, and what is the place of probability topic in mathematics curriculum. Afterwards, observations were made by monitoring teacher-student interactions and the lectures of the teachers for 4-6 course hours, inside and outside the classroom, until the completion of the probability subject. The observations were made because we wanted to ensure the consistency of what teachers said during the interviews.

Data Analysis

The interview records and observation notes were transferred to the computer and were converted to written texts in the first phase of analyzing the data. Afterwards, to reveal the PCK of the teachers about probability, data obtained from the preliminary interviews and the observations made during the courses were analyzed together using deductive analysis, in which the data are analyzed according to an existing framework through cross-case analysis (Patton, 2002, p.453). The data analysis took place according to the themes formed by the help of PCK components, i.e., content knowledge, curriculum knowledge, student knowledge, and knowledge of teaching methods and strategies. Furthermore, an inductive approach was adopted to analyze the data existing apart from those themes and to get the sub-themes for previously determined themes owing to the fact that the sub-themes were not determined in advance. The data were coded and similar or interrelated codes combined under the related themes. For example, the codes named “associated topics” and “prerequisite knowledge” have been united as “associated/ prerequisite concepts or topics”. Upon the formation of codes by two researchers independently, the codes were compared to check the consistency and coding reliability. In cases of different coding, the researchers discussed and came to a consensus. As a result of the analyses, a fifth theme (i. e., teacher beliefs) and a variety of sub-themes (e. g., knowledge of probability, horizontal curriculum knowledge, student development, etc.) have been emerged. It should also be noted that sub-themes were consistent with the existing literature (e. g., horizontal and vertical curriculum knowledge) and those sub-themes were named taking into consideration the existing literature.

While processing the data, pseudonyms were used to protect the privacy of teachers, and quotations from the teachers were made where necessary. In the quotations, punctuation marks were not used unless required, in terms of reflecting the context; words were not corrected; and the researchers tried to express the texts in the language of the speech.

Role of the Researcher

The first researcher graduated from the Department of Mathematics Teaching and pursues his doctoral education in curriculum and instruction. His master’s thesis and doctoral dissertation are about mathematics teaching, and he took courses related to mathematics teaching as well as curriculum development. He also took a course in qualitative study, and he has a book chapter resulting from qualitative research of his own. The second researcher is a lecturer in the Department of Mathematics Teaching and has published articles using qualitative design. She also lectures on mathematics focused on PCK at the doctorate level.

Within the present study, the instrument was created by two researchers while the interviews and observations were conducted by the first researcher. Both researchers carried out the data analysis.

Validity and Reliability of the Study

To ensure the validity in the collection and analysis of the study's data, triangulation was considered and data from the interviews were supported with the observations. In addition, the researcher prolonged the uptime by spending time with the participants during the breaks and after the course and tried to reach data saturation. The researcher presented his arguments, deducted from the observations, to the participants, and he discussed the reasons for teachers' in-class behaviors with them. In the study, where more than one data collection method was used, the data have been presented in a complementary way; participants were consulted about the appropriateness of the results reached during analysis.

The purpose of qualitative studies is not to generalize the results obtained; however, to ensure external validity of the study, the consistency of the created themes with those mentioned in the literature has been checked. In addition, research findings were interpreted and discussed regarding the data obtained from the study and the context of the research. Consequently, the findings and the results of this research can be generalized analytically for the cases similar to the ones discussed.

Regarding the study's reliability, the researchers combined all the data obtained from one of the cases in the study (from pre- and post-interview notes, observation notes, and the observation form) and have performed encoding. After encoding the data of the other two cases independently, they came together and compared generated codes. Differentiating codes were revised once more with a mutual exchange of ideas. In addition, to increase the reliability of the study, the whole process of data collection and analysis is provided in detail.

FINDINGS

In this study, where PCK of the teachers has been examined via deductive analysis of data obtained through teacher interviews and class observations, the findings are discussed under five themes (see Figure 1). An additional theme, teacher beliefs, has come out as a result of the codes. We wanted to present this theme together with the previously formed themes since the findings showed that the teacher beliefs strongly affect PCK.

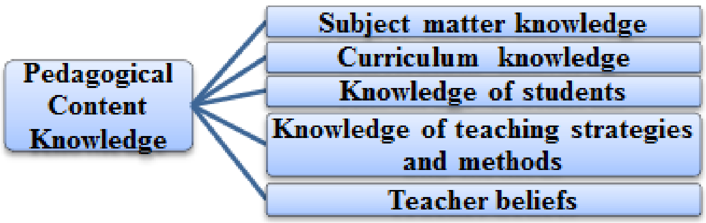

Figure 1: The themes related to the instruction of probability

PCK of Mathematics Teachers

There are five themes in this category that were generated as a result of analysis of preliminary interviews and class observations. These themes are (i) content knowledge, (ii) curriculum knowledge, (iii) knowledge of students, (iv) knowledge of teaching strategies and methods, and (v) teacher beliefs.

Subject-matter knowledge: Content knowledge of the participants is discussed under two sub-themes: (i) associated and prerequisite concepts or topics, which are the topics associated with probability and required to be known for teaching probability, and (ii) knowledge of probability (see Figure 2).

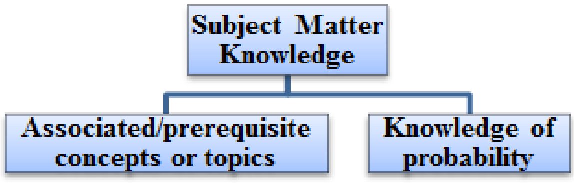

Figure 2: The sub-themes related to the subject-matter knowledge

Teachers have usually mentioned rational numbers, operations, and ratio-proportion topics as the associated and prerequisite concepts or topics of probability. In addition, one participant, Esin, stated sets; another participant, Lale, stated factorization, percentages, and numbers (odd, even, prime number, etc.) as prerequisite topics; and the third participant, Nihat, stated that it is necessary to learn the priority of operations before the concept of probability. In this context, all three teachers are different from each other in terms of prerequisite concepts or topics. However, the researchers observed that the teachers were not checking the knowledge of the students about these subjects before starting the lecture; they just reminded the students of these subjects when required during the course.

The other sub-theme discussed in terms of subject-matter knowledge is the knowledge that teachers possess about probability. Each of the three teachers feels inadequate in some of the concepts related to probability, a fact reflected in their teaching and their format of answering questions from students., Esin expressed her confusion of dependent-independent events and discrete and non-discrete events with the sentence "… *I sometimes confuse dependent-independent events, discrete and non-discrete events while trying to understand the question* …" In the lecture, while teaching dependent-independent events, Lale and Nihat gave "tossing a coin and rolling a die together" as an example of independent events, but when a student asked if that event would be dependent "in case of midair collision of the coin and dice," they answered in a way to pass the matter lightly. In addition, during the course, Lale had difficulty understanding the correct answers given by students if solved in different ways from what she showed.

One of the questions used during the interviews, which addressed subject-matter knowledge of the teachers, was regarding "the standing upright probability of a coin when tossed." During the preliminary interview, when the reactions of the teachers towards the students' questions about the probability of "the coin standing upright" were discussed, the teachers answered that they would consider it as an effort for "disrupting the lesson" and they wouldn't care (Esin) or they would answer it as "there is no chance for the coin to stand upright, it will not occur" (Lale and Nihat). When students asked the standing upright probability of a coin during the course, Esin stated that "this can only happen in movies" and did not focus on the question, whereas Lale and Nihat explained that the sum of the probability of getting a head (1/2) and tail (1/2) is already 1, hence there is no chance for the coin to stand upright and its probability is "0". Therefore, unfortunately, the teachers could not use this learning opportunity and could not associate theoretical and experimental probability with this example, which was included in the 8th grade mathematics curriculum.

The researchers observed that the subject-matter knowledge of Lale is lower compared to the other two teachers. She could not think of and interpret the concept of probability as a unique, standalone topic. Lale's statement of *"probability, may be or may not be [it is] not clear"* supports this idea, which makes us think that she did not configure this concept in her mind. She also thinks of probability as her weakness on the rationale of abstractness. In addition, Lale addresses probability, permutation, and combination, which are associated but different concepts, within the same body; she answered questions about the concept of probability by combining them with the concepts of permutation and combination.

Curriculum knowledge: The knowledge of the teachers about the curriculum is organized under two themes, which are vertical and horizontal curriculum knowledge (see Figure 3). Vertical curriculum knowledge is understood as the knowledge of mathematics teachers about the topics taught in the other grade levels, whereas horizontal curriculum knowledge is considered as the mathematical content knowledge of teachers at one grade level.

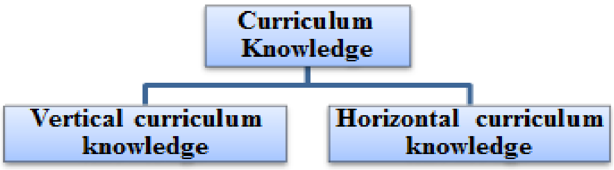

Figure 3: The sub-themes related to the curriculum knowledge

Regarding vertical curriculum knowledge, Esin and Nihat stated that the topic of probability starts at 6th grade, whereas Lale, although she was not sure, stated that it is taught at 5th grade. Also, all three teachers added that they didn't review the new program updated in 2013, and they are not aware of the distribution of topics, including probability, except in the grades that they are teaching. In fact, in the previous curriculum, the probability topic was given starting from 4th grade whereas with the updated program it is only taught at 8th grade. This fact indicates that the teachers are deprived of vertical curriculum knowledge.

Although the teachers' horizontal curriculum knowledge is relatively better than their vertical curriculum knowledge, it still seems incomplete. Esin and Nihat stated that they review the content of the topic the day before the course whereas Lale stated that since she teaches continuously, she doesn't prepare before the course, but she decides the examples to be solved in the classroom. These practices cause the teachers not to master both the subject and the sequence of the sub-concepts at the class level that they teach. The teachers' lack of horizontal curriculum knowledge was revealed during interviews when Esin did not remember the types of probability, Nihat stated that impossible and certain events were not taught in the previous grades and that students would learn them for the first time, and Lale thought that permutation, combination, and factorial concepts would also be given with probability.

Knowledge of students: Teachers' knowledge about students is discussed under three sub-themes: students' prior knowledge, their misconceptions and difficulties, and student development (See Figure 4). The prior knowledge sub-theme includes basic knowledge about probability that students should know in advance; the misconceptions and difficulties sub-theme express the difficulties that students encounter or that they may experience in the future and their misconceptions about probability; and the student development sub-theme shows the teachers' opinions about the conformance of this topic to the students' level of development and their individual differences.

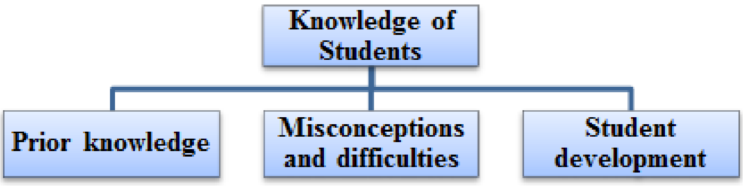

Figure 4: The sub-themes related to the knowledge of students

All three teachers stated that some prior knowledge is needed to learn probability effectively, and this prior knowledge is mentioned in association with ratios, proportions, percentages, fractions, and rational number concepts. In addition, the teachers think that students ought to be aware of basic concepts because of the introduction to probability taught in previous years, but due to the lack of vertical curriculum knowledge, they cannot be sure what kind of prior knowledge students have about probability. Although the teachers stated that the topics that will facilitate the teaching of probability and the basic concepts of probability should be known by the students, only Esin reminded the students about the basic concepts of probability at the beginning of the course.

Regarding students' misconceptions and difficulties associated with probability, all three teachers stated that students have difficulties in learning probability, but they could not express clearly what these challenges are. Esin thinks that all students can learn this topic and it should not be a challenge for them, and she expressed that students mostly confuse dependent-independent events with discrete and non-discrete events. Considering that Esin sometimes confuses these concepts, as mentioned in the subject-matter knowledge section, it can be said that she interpreted the difficulty that she has encountered as a difficulty encountered by students. Nihat and Lale stated that probability is a boring and abstract topic for the students and added that the difficulties that students have encountered are due to their lack of reasoning skills. However, the teachers did not give specific examples about the difficulties that students have experienced on the subject of probability. This fact can be interpreted as the teachers having tagged probability as a topic that "students will have difficulty in understanding." In addition, Nihat thinks that students preparing before the course and solving many examples will help them overcome challenges about probability, whereas Lale expressed that the difficulties of the students on this issue cannot be overcome. These expressions featuring overcoming students' challenges about probability can be shown as evidence that the teachers don't clearly know what students' difficulties are.

In terms of student development, Esin thinks that all students can learn probability without encountering any difficulty, which shows that she doesn't consider individual differences and the cognitive development level of the students. However, this view is probably due to teaching in a high-achieving school, as she herself also mentioned. Nihat stated that successful students understand probability better and that the topic requires higher-level cognitive skills, whereas Lale stated that students lack numerical intelligence. She associated students' underachievement in this topic with the shortcoming in their cognitive development:

Lale: [It is difficult for the students] because these students don't have numerical intelligence. Each of the students does not understand this topic at the same amount. In other words they hardly learn. I mean this is a difficult, airborne topic.

Knowledge of teaching strategies and methods: Teachers' knowledge about teaching strategies and methods is discussed under two sub-themes: association of probability with daily life and use of examples in teaching probability (see Figure 5). The first sub-theme deals with the cases students may encounter in daily life whereas the second involves the illustrations used to teach probability effectively.

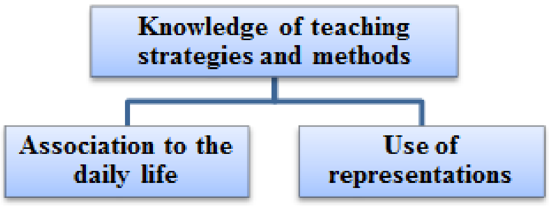

Figure 5: The sub-themes related to the knowledge of teaching strategies and methods

The teachers stated that association of the topic with daily life during the lecture may contribute to students' understanding of probability, but the examples given about daily life are too classical. The examples about the teachers' association of probability with daily life are usually tossing a coin, rolling a die, and drawing cards experiments. Unlike these examples, Esin addressed the relationship between probability and daily life in terms of "chance games" and reflected this idea in her lecture; Nihat used the case of "different cars to win a race" for subjective probability, whereas Lale showed the prediction of "the probability of rain" as an example. On the other hand, Esin, identifying probability concepts with possibility, considers probability as a fact that is encountered continuously in daily life, expressing it as "*Anything can happen anytime, anywhere.*"

Regarding effective instruction on the topic, the teachers stated that the course should be visualized and enriched with a variety of examples. Regarding the use of materials (concrete representations), Esin brought a die made of cardboard to the classroom and a few students made experiments, whereas Lale attempted to ensure the students' understanding of the types of probability by presenting an activity on the computer. Nihat told that he could bring bags and balls to the class and perform a ball pulling experiment, but he didn't use any material in the classroom. However, Nihat has showed the results of tossing a coin successively on the board with the tree diagram as a visual representation (Figure 6a). Lale has also used a visual representation: she showed the outputs of the experiment of tossing a coin and rolling a die concurrently by drawing arrows in two different ways (Figure 6b and 6c).

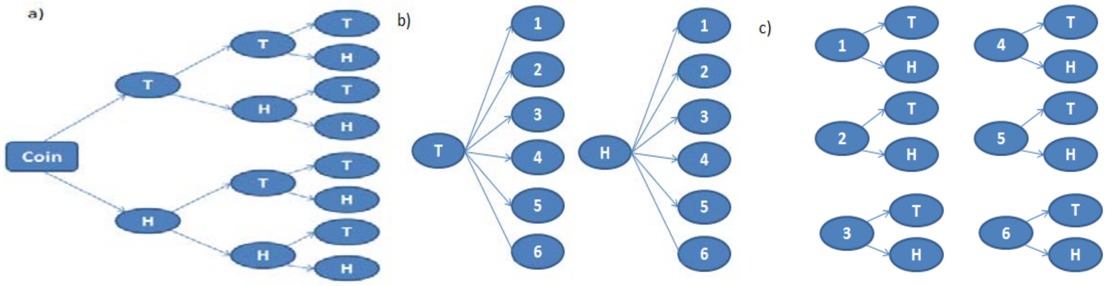

Figure 6: The representations used by Nihat and Lale

During preliminary interviews, the teachers expressed that students encountered difficulties with this topic, but when they were asked about the methods and strategies that they would use in class, they didn't identify anything other than associating with daily life and using material. Indeed, this situation is reflected in their lecturing process as well; association of probability with daily life and use of materials during lecture were not used effectively by the teachers. Regarding the examples, the same number appeared on the upper side of the die that Esin used, and the video activity used by Lale was not found to be interesting by the students; on the contrary, it has been described as "unrealistic" by the students since it was a narrative depicting a sultan and his daughter. The teachers didn't use alternative approaches to ensure better understanding of students; , they lectured using traditional methods, such as plain instruction and question-and-answers.

Teacher Beliefs: Although the teacher beliefs were not among the components we wanted to examine, this theme existed during the analyses in connection with almost all the other themes. Hence, we felt the need for including such a theme affecting the other components of PCK (Magnusson et al., 1999). The findings about teachers' beliefs are discussed under four sub-themes: beliefs regarding himself/herself, the subject, the curriculum, and the students (see Figure 7). Beliefs of the teachers towards themselves indicate self-efficacy beliefs about teaching probability whereas their beliefs towards the subject express their beliefs towards the necessity of teaching probability to the students and how much it should be taught. In addition, teachers' beliefs about the curriculum indicate their beliefs about the structure of the curriculum together with the beliefs towards the class level appropriate for teaching probability and its content, whereas their beliefs towards students represents their beliefs towards the students' competencies for learning this topic.

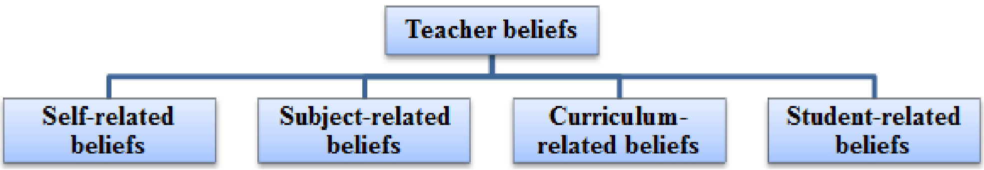

Figure 7: Sub-themes related to the teacher beliefs

Esin, despite finding herself quite sufficient in teaching probability, believes that there is room for improvement and thus expressed that she feels incomplete. However, she believes that she teaches the topic well and she can teach probability to the students at all levels. The overconfidence of Esin leads her to see the topic simpler than it is and lets her think that there is no possibility for students to have various difficulties in understanding the subject. Thus, her beliefs also affect the students; a teacher thinking that the topic is simple prevents students from asking about the points that they don't understand. In fact, only the same few students answered the questions asked during the course; moreover, the teacher couldn't get the right answer when she asked another student, which set an example for the case. In addition, during the course, Esin attempted to give an example that was not included in the course book, then abandoned the example because the setup was wrong. Contrary to the overconfident Esin, Lale sees probability, which she described as “a topic of nuisance,” as her weakness and expressed that she doesn’t enjoy teaching this topic. In addition, she thinks that the difficulty that she encounters while teaching the topic is caused by the inability to express herself. Lale stuck to the book during the course, and when a student answered a question incorrectly, she just gave the right answer without questioning the reasons of the wrong one, which shows that her self-efficacy beliefs about teaching probability are low. Nihat is quite self-confident from teaching the subject for years, and he didn’t make any statement about feeling incompetent on this subject. During the courses, Nihat attempted to answer all the students’ questions and encouraged students who were encountering difficulties in understanding, which shows that his self-efficacy belief was reflected in the course in a balanced way.

Regarding the beliefs towards the topic, Esin, who enjoys teaching probability, indicated that the topic was interesting because she mostly associated it with chance games. She added that she likes probability because the use of material during the course is simpler compared to the other topics. On the other hand, Lale stated that probability is an abstract topic and many concepts on the topic “remain in the air” in the sense that “those concepts indicate the possibility.” For this reason, she thinks that there is no need to explain the topic to the students in such detail; it can be given “superficially.” In addition, she stated that the majority of the mathematics teachers that she knows think like her, and she supports her thoughts on this subject in this way. She also said that she enjoys the subjects requiring more operations, such as exponential or root numbers, more than the ones remaining in “suspense,” such as probability. Nihat thinks that probability is not different than other topics, and he didn’t express any positive or negative belief towards the topic:

Lale: The probability is very confusing after a while; if you upload more stuff it explodes in the hands.

Regarding the beliefs towards the curriculum, Esin stated that the probability coverage of secondary school is “almost at high school level” and found it intensive, but she thinks that the coverage of 8^{th} grade is appropriate and the topics at this level can be taught if there is enough time. However, since students confuse some of the concepts, she proposed teaching these confused concepts at different grade levels. Nihat believes that the coverage of probability can be narrowed because it will be discussed again in the following years; in addition, he pointed out that the topics of permutation and combination should be given with probability in the curriculum. Lale also stated that the coverage of the topic should be narrowed at the secondary school level; furthermore, she doesn’t like the spiral structure of the curriculum because she doesn’t know what and how much has been taught in the previous years, and she thinks that teaching the topic in pieces at various grades is boring.

Esin, who stated that students can comfortably learn this topic in spite of some difficulties, has a positive approach towards the topic. She sees students like herself, and she thinks that students enjoy learning probability. Nihat stated that the topic is comfortably understood by successful students and that students with good reasoning skills will not encounter any difficulty in learning probability. On the other hand, Lale, who emphasized that the topic is abstract, expressed that it is difficult for the students to understand this topic and it doesn’t appeal for this age group. In addition, she thinks that because all students are not at the same level of numeric intelligence, they don’t understand the topic at the same level:

Lale: Do you know there is a proverb, “if the God doesn’t give what can Mahmut do”? [A Turkish proverb meaning that even after the God did not want, the power of sultan is not enough to make something.]

DISCUSSION AND CONCLUSION

This study tried to examine the pedagogical content knowledge of the secondary school teachers related to the probability and the findings have been presented above. The results have been discussed here and it is important to remind the readers that the discussion just based on the results of the present study and it is aimed not to overgeneralize the findings.

The main finding from this research in which the probability-related PCK of secondary school teachers has been examined is that the most important factors impacting teachers’ PCK are teacher beliefs rather than their experiences, and consequently, the teachers' efforts to improve themselves. In particular, the teacher who has positive beliefs towards herself and the topic has good content knowledge and possesses the self-confidence to teach the topic effectively to students. The teacher who feels inadequate about probability and doesn't like this topic thinks that she cannot teach the topic effectively and believes that her point of view about probability is also shared by students. It can be said that the source of the differences on the practices of these teachers, who have similar seniority, is their beliefs, particularly towards themselves and the topic, as well as towards the students and curriculum. The teacher with the most seniority sees probability equivalent to other topics; he doesn't address this topic that students encounters difficulties in a special way. Also, due to his seniority, he was quite self-confident in terms of teaching the topic, and this let him feel more comfortable than the other teachers during the lecture. Parallel to the findings of the study, there are many studies demonstrating that students' beliefs are associated with the components of PCK (Knight & McNeill, 2011; Kumar & Subramaniam, 2012) and with the teacher practices (Aguirre & Speer, 2000; Ernest, 1989; Peterson, Fennema, Carpenter, & Loef, 1989; Richardson, 1996; Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1992; Yero, 2002). Furthermore, Magnusson et al. (1999) integrated the notion of belief to the PCK framework, which also matches up with the findings of this study.

The review of teachers' content knowledge showed that the content knowledge of Esin, who improves herself and has positive beliefs, is better than other teachers. However, all three teachers' content knowledge about probability is insufficient. During the observations, it was seen that the teachers' content knowledge has been directly reflected in their teaching processes. Thus, content knowledge is a determinant of teachers' effective probability teaching (Ball, 1990b; Halim & Meerah, 2002). In addition to their lack of content knowledge, their knowledge about teaching methods and strategies is also insufficient. Regarding the association of probability with daily life, the teachers were usually stuck in the die-coin pattern, but various examples such as raining, predicting the winner of a car race, and associating probability with chance games were also observed. In this context, teachers have not shown additional effort concerning teaching probability effectively; they have chosen a plain lecturing method during the course, and they didn't use alternative approaches such as cooperative learning. This may be the result of the national exam which the students are supposed to take. The teachers have to teach a variety of subjects, and they have a limited period. Additionally, probability requires higher-level cognitive skills compared to other topics, and students encounter difficulties in terms of reasoning skills for probability (Bulut, 1994; Fischbein & Gazit, 1984; Greer, 2001; Kustos, 2010). Thus, for teaching this subject effectively, a learning environment where students are more active and express their ideas clearly, rather than the transfer of information from teacher to student, is required. Within the study, all three teachers directly focused on students' answers being right or wrong without questioning the reasoning behind it, which reduces the likelihood of students' conceptual learning of this topic. In addition, teachers also lack curriculum and student knowledge. The teachers, who lack vertical curriculum knowledge, also have shortcomings in terms of horizontal curriculum knowledge, which cause them to not be sure about students' prior knowledge. It can be said that curriculum knowledge impacts student knowledge with regard to the prior knowledge. In addition, teachers get an idea about the content of the course by reviewing the content and objectives that they will teach before the course; thus, their shortcomings about the curriculum don't mean anything to them. Therefore, it is not so important for teachers to have vertical curriculum knowledge (Baştürk & Dönmez, 2011).

Research investigating the probability-related PCK of mathematics teachers is quite limited. The literature we have researched is theoretical, published as proceedings or conducted with pre-service teachers. There were just two research studies examining the mathematics teachers' probability-related PCK among the ones we have researched, one of which was conducted through a questionnaire (Brijlall, 2014) and the other almost two decades ago (Swenson, 1997). Hence, we decided to conduct qualitative research, as there are not enough studies focusing on this issue, though there are a variety of studies putting forth the probability-related misconceptions and difficulties of both students and teachers. The nature of PCK teachers have about probability would affect such problems directly. Although we did not examine the teachers' attitudes towards the instruction of probability straight-forwardly, it has been seen that the teachers had an awareness about the PCK components, and they started to question themselves. Hence, the findings of the present study have implications for mathematics education, teacher preparation, in-service training, and curriculum development.

According to the findings of the present study, PCK of secondary school mathematics teachers about probability is insufficient. Although lack of teaching materials and teacher-centered lessons processed by traditional methods (Castro, 1998; Gürbüz, 2007) are among the causes of the difficulties in teaching probability, the main reason is that a great majority of mathematics teachers don't have the quality required to teach probability effectively (Brijlall, 2014; Bulut, 2001; Chick & Baker, 2005). Therefore, in Turkey, in order to teach probability effectively, which is difficult to understand by the students as well as by the teachers (Bulut, 1994; Işık et al., 2011; Sezgin-Memnun, 2008), the necessary in-service training should be given to teachers. In addition, measures should be taken to eliminate shortcomings of student teachers, who have difficulty in the basic concepts of probability, as was revealed in many studies conducted in Turkey (Işık et al., 2011). In addition, considering that the majority of the studies on teaching probability were conducted with student teachers (Bulut, 2001; Işık et al., 2011) and the studies featuring teachers' PCK on this topic are insufficient, the findings of the present study show that new studies are needed in this area.

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